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Hall Ticket Number:

CE/EC/ME111 (R20)

B.TECH. DEGREE EXAMINATION, DECEMBER-2024

Semester I [First Year] (Supplementary)

MATHEMATICS - I

Time: Three hours Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:
   - (a) State symmetry of beta function CO1
   - (b) Compute $\Gamma\left(-\frac{5}{2}\right)$. CO1
   - (c) Find the area enclosed by the parabolas $x^2 = y$ and $y = x$. CO1
   - (d) Write the geometrical interpretation of Lagranges mean value theorem. CO2
   - (e) Show that $\sum_{n-1}^{\infty} \sin\left(\frac{1}{n}\right)$ is divergent. CO2
   - (f) State Rolle's theorem. CO2
   - (g) If $\bar{f} = xy^2\bar{i} + 2x^2\,yz\bar{j} - 3yz^2\bar{k}$ find $\mathrm{div}\bar{f}$ at $(1, -1, 1)$. CO3
   - (h) If $x = r\cos\theta$, $y = r\sin\theta$ Show that $\frac{\partial r}{\partial x} = \frac{\partial x}{\partial r}$. CO3
   - (i) Find the Fourier sine series of $f(x) = x^2$ in $[0, \pi]$. CO3
   - (j) Find the rank of a matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$. CO4
   - (k) If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then find the eigen values of A^T. CO4
   - (l) If $\lambda = 1, 2, 3$ are the eigen values of a matrix $A_{3\times 3}$, then find the eigen values of A^{-1}. CO4
   - (m) State cayley-Hamilton theorem. CO4

(n) Find the value of 'k' such that the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & k & 7 \\ 3 & 6 & 10 \end{bmatrix}$ is 2. CO4

UNIT – I

2. (a) Show that the equation of the evolute of the parobala $x^2 = 4ay$ is $4(y - 2a)^3 = 27ax^2$. (7M) CO1

(b) Prove that $\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\pi}{4}$. (7M) CO1

(OR)

3. (a) Prove that $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta \, d\theta = \frac{1}{2}\beta(m,n)$ (7M) CO1

(b) Find the surface area of revolution generated by revolving the curve $x = y^3$ from $y = 0$ to $y = 2$. (7M) CO1

UNIT – II

4. (a) Use Taylor's series to expand $2x^3 + x^2 + x + 1$ in powers of $(x - 1)$. (7M) CO2

(b) Verify Rolle's theorem for the function $\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in (a, b), where $a > 0$. (7M) CO2

(OR)

5. (a) Examine the convergence of the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + - - -$ (7M) CO2

(b) Test for convergence of $\sum\left[\sqrt{n^3+1} - \sqrt{n^3}\right]$ (7M) CO2

UNIT – III

6. (a) If $r^2 = x^2 + y^2 + z^2$ and $u = r^m$ and then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$ (7M) CO3

(b) Obtain the Fourier series for the function $f(x) = x \sin x$, $0 < x < 2\pi$. (7M) CO3

(OR)

7. (a) Find $\text{div}\,\bar{f}$ where $\bar{f} = r^n\,\bar{r}$. Find n if it is solenoidal. (7M) CO3

(b) Find the maximum and minimum values of $f(x, y) = x^2y + xy^2 - axy$. (7M) CO3

UNIT – IV

8. (a) Reduce the matrix $\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$ into normal form and hence find its rank. (7M) CO4

(b) Determine the rank, nature, index and signature of the quadratic form $2xy + 2yz + 2zx$ by reducing into canonical form using orthogonal transformation. (7M) CO4

(OR)

9. (a) Test for consistency and hence solve $x + y + 2z = 4$, $2x - y + 3z = 9$, $3x - y - z = 2$. (7M) CO4

(b) Determine the modal matrix P for $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ and hence diagonalize A. (7M) CO4

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CE/EC/ME111 (R20)

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Hall Ticket Number:

CE/EC/ME111 (R20)

B.TECH. DEGREE EXAMINATION, APRIL-2024

Semester I [First Year] (Supplementary)

MATHEMATICS-I

Time: Three hours — Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:

(a) What is the value of $\int_0^\infty e^{-x}dx$? CO1

(b) Define gamma function. CO1

(c) Write different kinds of improper integrals. CO1

(d) State Lagrange's mean value theorem. CO2

(e) Write Maclaurin's series expansion of $\tan^{-1}x$ if it exists. CO2

(f) Determine the nature of the sequence $\left\{\frac{1+(-1)^n}{n}\right\}_{n=1}^{\infty}$. CO2

(g) Write the necessary condition for convergence of a series. CO3

(h) Define half-range sine series. CO3

(i) If $x^y = y^x$ is the implicit relationship between x and y, then find $\frac{dy}{dx}$. CO3

(j) Define saddle point of a function two variables. CO4

(k) Is $\overline{f} = (y+z)\overline{i} + (z+x)\overline{j} + (x+y)\overline{k}$ irrotational vector? CO4

(l) Write elementary row transformations on a matrix. CO4

(m) State Cayley-Hamilton theorem. CO4

(n) When we say the system of linear equations are consistent? CO4

UNIT – I

2. (a) Discuss the convergence of $\int_0^3 \frac{1}{x^2-3x+2}dx$. (7M) CO1

(b) Evaluate the integral $\int_0^1 x^5\left[\ln(1/x)\right]^3 dx$. (7M) CO1

(OR)

3. Determine the volume and surface area of the solid generated by the revolution of the lemniscate $r^2 = a^2\cos 2\theta$ about the perpendicular line. CO1

UNIT – II

4. (a) Verify Rolle's theorem for $f(x)=\ln\left(\frac{x^2+ab}{x(a+b)}\right)$ in (a,b) where a > 0. (6M) CO2

(b) Expand sin x in powers of $x-\pi/2$ and hence find the value of $\sin 91^o$ correct to 4 decimal places. (8M) CO2

(OR)

5. (a) Discuss the convergence of $\sum_{n=1}^{\infty}\frac{(\ln n)^2}{n^{3/2}}$. (7M) CO2

(b) Examine the convergence of $\sum_{n=1}^{\infty}\frac{4.7...(3n+1)x^n}{1.2...n}$. (7M) CO2

UNIT – III

6. (a) Find the half-range Fourier cosine series for $f(x)=(x-1)^2$ in $0<x<1$. (6M) CO3

(b) If $x^x y^y z^z = e$, then prove or disprove that $\frac{\partial^2 z}{\partial x\partial y}=-\left[x\ \ln(ex)\right]^{-1}$ at $x=y=z$. (8M) CO3

(OR)

7. (a) Find the points on the surface $z^2 = xy+1$ that are nearest to origin. (9M) CO3

(b) Find the angle of intersection of the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the point $(2,-1,2)$. (5M) CO3

UNIT – IV

8. (a) Determine a, b, c so that A is orthogonal, where $A=\begin{bmatrix}0 & 2b & c\\ a & b & -c\\ a & -b & c\end{bmatrix}$. (4M) CO4

(b) Show that the system of equations $2x-2y+z=\lambda x,\ 2x-3y+2z=\lambda y,\ -x+2y=\lambda z$ can possess a non-trivial solution only if $\lambda=1,\ \lambda=-3$. Obtain the general solution in each case. (10M) CO4

(OR)

9. Reduce the quadratic form $5x^2+26y^2+10z^2+4yz+6xy+14zx$ to the canonical form by using diagonalization method, and hence find its rank, nature, index and signature. CO4

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CE/EC/ME111 (R20)

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Hall Ticket Number:

CE/EC/ME111 (R20)

B.TECH. DEGREE EXAMINATION, JANUARY-2024

Semester I [First Year] (Regular & Supplementary)

MATHEMATICS-I

Time: Three hours   Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:
   - (a) Define evolute. CO1
   - (b) Define beta function. CO1
   - (c) Write the relation between beta and gamma functions. CO1
   - (d) State Lagrange's mean value theorem. CO2
   - (e) Write Maclaurin's series expansion of f(x). CO2
   - (f) State Raabe's test for convergence. CO2
   - (g) Define Irrotational vector. CO3
   - (h) Define stationary point of a function. CO3
   - (i) Define gradient of a function. CO3
   - (j) Define minor of a matrix. CO4
   - (k) State Rank-Nullity theorem. CO4
   - (l) Write the eigen values of A^2 if $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 2 \end{bmatrix}$. CO4
   - (m) State Cayley-Hamilton theorem. CO4
   - (n) Define canonical form of a quadratic form. CO4

UNIT – I

2. (a) Prove that the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $ax^{2/3} + by^{2/3} = (a^2 - b^2)^{2/3}$. (7M) CO1

   (b) Show that $\int_0^\infty x^4 e^{-x^2} dx = \frac{3\sqrt{\pi}}{8}$. (7M) CO1

(OR)

3. (a) Prove that $\int_0^1 (x-a)^m (b-x)^n dx = (b-a)^{m+n+1}\beta(m+1, n+1)$. (7M) CO1
   (b) Find the volume of a solid generated by revolving the portion of the parabola $y^2 = 4ax$ cut off by its latus-rectum about the axis of the parabola. (7M) CO1

UNIT – II

4. (a) Verify Rolle's theorem for the function $\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in (a, b), a > 0, b > 0. (7M) CO2
   (b) Find the Taylor's series expansion of sin x in powers of $(x - \frac{\pi}{4})$. (7M) CO2

(OR)

5. (a) Test for convergence of the series $\sum \frac{1}{n} \sin \frac{1}{n}$. (7M) CO2
   (b) Test for convergence of the series $\sum \frac{n+1}{n} x^{n-1}$. (7M) CO2

UNIT – III

6. (a) Find the Half-range sine series for the function $f(x) = x(\pi - x)$ in the range $(0, \pi)$ and hence deduce that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \cdots = \frac{\pi^3}{32}$. (7M) CO3
   (b) Find the minimum value of $x^2 + y^2 + z^2$ if $x + y + z = 3a$. (7M) CO3

(OR)

7. (a) Find the directional derivative of $f = xy + yz + zx$ in the direction of vector $\bar{i} + 2\bar{j} + 2\bar{k}$ at the point $(1, 2, 0)$. (7M) CO3
   (b) Show that $\nabla^2(r^m) = m(m+1)r^{m-2}$ (7M) CO3

UNIT – IV

8. (a) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by reducing it into normal form. (7M) CO4
   (b) Prove that the following system of equations are consistent and solve them 3x + 3y + 2z = 1; x + 2y = 4; 10y + 3z = –2; 2x – 3y – z =5. (7M) CO4

(OR)

9. (a) Using Cayley-Hamilton theorem, find the inverse of the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. (7M) CO4
   (b) Reduce the following quadratic form to canonical form $2x^2 + 2x^2 + 2x^2 - 2xy - 2yz - 2zx$. (7M) CO4

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CE/EC/ME111 (R20)

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Hall Ticket Number:

CE/EC/ME111 (R20)

B.TECH. DEGREE EXAMINATION, JUNE-2023

Semester I [First Year] (Supplementary)

MATHEMATICS - I

Time: Three hours — Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:
   - (a) Define Gamma function. CO1
   - (b) Write the formula for volume of solid of revolution of a curve y = f(x) about x-axis form x = a to x = b. CO1
   - (c) Determine $\beta\left(\frac{5}{2}, \frac{3}{2}\right)$. CO1
   - (d) Explain why mean value theorem does not hold for $f(x) = x^{\frac{2}{3}}$ in $[-1, 1]$ CO2
   - (e) Discuss the convergence of series $\sum_{n=0}^{\infty} \frac{1}{3^n}$ CO2
   - (f) State Lagrange's Mean value theorem. CO2
   - (g) Explain geometrical interpretation of $\nabla\phi$ CO3
   - (h) Find $\frac{\partial^2 u}{\partial x \partial y}$ for the function $u = \tan^{-1}\left(\frac{x}{y}\right)$ CO3
   - (i) Write the Dirichlet's conditions for the existence of Fourier series. CO3
   - (j) Define rank of a matrix and find the rank of identity matrix of order n. CO4
   - (k) If A = $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then find the eigen values of A^T. CO4
   - (l) State Cayley-Hamilton theorem. CO4
   - (m) Find the rank of a matrix $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$. CO4
   - (n) If the Eigen values of matrix A_{3X3} are 1, 2, 3 then find det(A). CO4

UNIT – I

2. (a) Find the evolute of the asteroid $x = acos^3\theta$, $y = asin^3\theta$. (7M) CO1

   (b) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\, dx \times \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}} = \pi$ (7M) CO1

(OR)

3. (a) Prove that the evolute of the curve x = a(cosθ + θsinθ), y = a(sinθ – θcosθ) is a circle. (7M) CO1

   (b) Prove that $\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\, d\theta = \frac{1}{2}\beta(m,n)$ (7M) CO1

UNIT – II

4. (a) Expand $x^3 - 2x^2 + x - 1$ in powers of (x – 1) using Taylor's series. (7M) CO2

   (b) Find the nature of the series $\frac{1}{2}x + \frac{1.2}{2.5}x^2 + \frac{1.2.3}{2.5.8}x^3 + ---- (x > 0)$ (7M) CO2

(OR)

5. (a) Test for convergence of the series $\sum_{n=1}^{x} \frac{1}{x^n + x^{-n}}$, $x > 0$. (7M) CO2

   (b) Calculate approximately $\sqrt[5]{245}$ by using Lagrange's mean value theorem. (7M) CO2

UNIT – III

6. (a) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. (7M) CO3

   (b) Find the directional derivative of $x^2yz + 4xz^2$ at the point (1, –2, –1) in the direction of the normal to the surface $xlogz - y^2$ at (–1, 2, 1). (7M) CO3

(OR)

7. (a) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational. (7M) CO3

   (b) Find the half-range cosine series for the function $f(x) = x(2-x),\ 0 \le x \le 2$ and hence find sum of series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ----$ (7M) CO3

UNIT – IV

8. (a) Find the value of λ for which the equations 3x – y + 4z = 3; x + 2y – 3z = –2; 6x + 5y + λz = –3 will have infinite number of solutions and solve them with that λ value. (7M) CO4

   (b) Find the value of K if the rank of $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$ is 2. (7M) CO4

(OR)

9. (a) Diagonalise the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ and hence find A^3. (7M) CO4

   (b) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A. (7M) CO4

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CE/EC/ME111 (R20)

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Hall Ticket Number:

CE/EC/ME111 (R20)

B.TECH. DEGREE EXAMINATION, MARCH-2023

Semester I [First Year] (Regular & Supplementary)

MATHEMATICS - I

Time: Three hours Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:
   - (a) Define Evolute. CO1
   - (b) Investigate the convergence of $\int_0^1 \frac{1}{1+x^2}dx$. CO1
   - (c) Prove that $\Gamma(1)=1$. CO1
   - (d) State Lagrange's mean value theorem. CO2
   - (e) Investigate the nature of $\sum_{n=1}^{\infty}\left(\frac{2022}{2021}\right)^n$ CO2
   - (f) State the comparison test for convergence. CO2
   - (g) Write Parseval's formula. CO3
   - (h) Find $\frac{dy}{dx}$ for $f(x,y)=x^2+y^2$ CO3
   - (i) Give the necessary condition for a function $f(x,y)$ have maxima or minima. CO3
   - (j) When do you say the vector point function $\vec{F}$ is irrotational? CO4
   - (k) Define rank of a matrix. CO4
   - (l) What do you mean by a system is said to be consistant? CO4
   - (m) Find eigen values of a matrix $\begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$ CO4
   - (n) Define canonical form of a quadratic form. CO4

UNIT – I

2. (a) Find the evolute of the curve $y^2 = 4ax$ (7M) CO1

(b) Prove that

$$\int_a^b (x-a)^m (b-x)^n \, dx = (b-a)^{m+n+1} \beta(m+1, n+1)$$ (7M) CO1

(OR)

3. (a) Prove that $\Gamma(n)\Gamma(1-n) = \dfrac{\pi}{\sin n\pi}$ (7M) CO1

(b) Find the area of the surface generated when the loop of the curve $9ay^2 = x(3a-x)^2$ revolves about the x-axis. (7M) CO1

UNIT – II

4. (a) Verify Rolle's theorem for the function $\ln\left[\dfrac{(x^2+ab)}{(a+b)x}\right]$ in [a, b], where a > 0. (7M) CO2

(b) Obtain the Maclaurin's series expansion of $\log_e(1+x)$ (7M) CO2

(OR)

5. (a) Test for convergence of the series $\dfrac{1}{1.3.5} + \dfrac{2}{3.5.7} + \dfrac{3}{5.7.9} + \ldots$ (7M) CO2

(b) Test for convergence of the series $1 + \dfrac{x}{2} + \dfrac{x^2}{5} + \dfrac{x^3}{10} + \ldots + \dfrac{x^n}{n^2+1} + \ldots$ (7M) CO2

UNIT – III

6. (a) Prove that $x^2 = \dfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \dfrac{\cos nx}{n^2}, \ -\pi < x < \pi$ and hence show that $\sum_{n=1}^{\infty}\dfrac{1}{n^2} = \dfrac{\pi^2}{6}$. (7M) CO3

(b) Investigate the maxima and minima of the function $f(x) = x^3 y^2 (1-x-y)$. (7M) CO3

(OR)

7. (a) Find the directional derivative of $\phi = xy + yz + zx$ at A in the direction of $\overrightarrow{AB}$ where $A = (1, 2, -1)$, $B = (1, 2, 3)$. (7M) CO3

(b) Define divergence and curl of a vector point function and give examples of each. (7M) CO3

UNIT – IV

8. (a) Find the rank of the matrix $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ by reducing into normal form. (7M) CO4

(b) Determine the value of λ for which the following set of equations may possesses non trivial solution $3x_1 + x_2 - \lambda x_3 = 0,\ 4x_1 - 2x_2 - 3x_3 = 0,\ 2\lambda x_1 + 4x_2 - \lambda x_3 = 0$ also find the solution for each λ. (7M) CO4

(OR)

9. (a) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 3 & 2 \\ 2 & 4 & 3 \end{bmatrix}$. (7M) CO4

(b) Reduce the quadratic form $x^2 + 3y^2 + 3z^2 - 2yz$ into canonical form. (7M) CO4

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CE/EC/ME111 (R20)

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Hall Ticket Number:

CE/EC/ME111(R20)

B.TECH. DEGREE EXAMINATION, OCTOBER-2021

Semester I [First Year] (Supplementary)

MATHEMATICS-I

Time: Three hours — Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:
   - (a) Define evolute. — CO1
   - (b) Investigate the convergence of $\int_0^1 \frac{1}{x^2}dx$ — CO1
   - (c) Define Gamma function. — CO1
   - (d) State Taylor's series. — CO2
   - (e) Is the series $1-\frac{1}{5}+\frac{1}{5^2}-\frac{1}{5^3}+\ldots\infty$ convergent or divergent — CO2
   - (f) State the nth root test. — CO2
   - (g) Find the half range sine series for $f(x)=x^2$ in $(0,1)$ — CO3
   - (h) Evaluate $\lim_{(x,y)\to(1,5)} \frac{xy}{x+y}$ — CO3
   - (i) Define the stationary point of a function. — CO3
   - (j) Define divergence of a vector point function. — CO4
   - (k) Find the product of the eigen values of the matrix $\begin{bmatrix} 2 & 18 & 20 \\ 0 & 4 & 19 \\ 0 & 0 & -1 \end{bmatrix}$ — CO4
   - (l) Define eigen vector of a matrix. — CO4
   - (m) Define the canonical form of a quadratic form. — CO4
   - (n) Define normal form of a matrix. — CO4

UNIT – I

2. (a) Find the envelope of a system of concentric and coaxial ellipses of constant area. (7M) CO1

   (b) Show that $\beta\left(m,\frac{1}{2}\right)=2^{2m-1}\beta(m,m)$ (7M) CO1

(OR)

3. (a) Show that $\int_0^\infty \frac{x^4}{4^x}dx=\frac{\Gamma(5)}{(\log x)^5}$ (7M) CO1

   (b) Prove that the evolute of the ellipse b$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=(a^2-b^2)^{\frac{2}{3}}$ (7M) CO1

UNIT – II

4. (a) Verify Rolle's theorem for $\frac{\sin x}{e^x}$ in $(0,\pi)$ (7M) CO2

   (b) Expand $\log_e x$ in powers of $(x-1)$ (7M) CO2

(OR)

5. (a) Test for convergence of the series $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+...\infty$ (7M) CO2

   (b) Test for convergence of the series $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+\sqrt{n+1}}$ (7M) CO2

UNIT – III

6. (a) Find the half range cosine series for $f(x)=x^2$ in $(0,\pi)$ (7M) CO3

   (b) In a plane triangle find the maximum value of the function $\cos A\cos B\cos C$ (7M) CO3

(OR)

7. (a) Show that $\nabla\left[\frac{f(r)}{r}R\right]=\frac{1}{r^2}\frac{d}{dr}\left[r^2 f(r)\right]$ (7M) CO3

   (b) Find the directional derivative of $x^2yz+4xz^2$ at the point $(1,-2,1)$ in the direction of the vector $2\vec{i}-\vec{j}+2\vec{k}$ (7M) CO3

UNIT – IV

8. (a) Find the rank of the matrix by reducing it to the normal form given (7M) CO4

   $A=\begin{bmatrix}1&2&3&4\\2&3&2&4\\1&-1&1&-1\end{bmatrix}$

   (b) Test for consistency $4x-2y+6z=8, x+y-3z=-1, 15x-3y+9z=21$ (7M) CO4

(OR)

9. (a) Verify Cayley-Hamiton theorem find the inverse of the matrix. (7M) CO4

   $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$

   (b) Reduce the quadratic form $8x^2+7y^2+3z^2-8yz+4zx-12xy$ to canonical form. (7M) CO4

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Hall Ticket Number:

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CE/EC/ME111(R20)

B.TECH. DEGREE EXAMINATION, JULY-2021

Semester I [First Year] (Regular)

MATHEMATICS-I

Time: Three hours | Maximum Marks: 70

Answer Question No.1 compulsorily. (14 x 1 = 14)
Answer One Question from each unit. (4 x 14 = 56)

1. Answer the following:
   - (a) Define Involute. CO1
   - (b) Investigate the convergence of $\int_0^1 \frac{1}{x} dx$ CO1
   - (c) Define Beta function. CO1
   - (d) State Maclaurian's series. CO2
   - (e) Is the series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+...\infty$ convergent or divergent. CO2
   - (f) State the Ratio test. CO2
   - (g) Find the half range sine series for $f(x)=x$ in $(0,1)$ CO3
   - (h) Evaluate $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$ CO3
   - (i) Define the stationary point of a function. CO3
   - (j) Define curl of a vector point function. CO3
   - (k) Find the product of the eigen values of the matrix. CO4

     $$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 2 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$
   - (l) Define eigen values of a matrix. CO4
   - (m) Define the canonical form of a quadratic form. CO4
   - (n) Define normal form of a matrix. CO4

UNIT – I

2. (a) Prove that the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $(ax)^{\frac{2}{3}}-(by)^{\frac{2}{3}}=\left(a^2+b^2\right)^{\frac{2}{3}}$ (7M) CO1

(b) Show that $\int_0^1 y^{q-1}\left[\log\frac{1}{y}\right]^{p-1}dy=\frac{\Gamma(p)}{q^p}$ (7M) CO1

(OR)

3. (a) Prove $\beta(m,n)=2\int_0^{\pi/2} sin^{2m-1}\theta\, cos^{2n-1}\theta\, d\theta$ (7M) CO1

(b) Find surface area of the solid formed by the revolution of $y^2=4ax$ about its axis by the arc. (7M) CO1

UNIT – II

4. (a) Verify Lagrange's mean value theorem for $(x+2)^3(x-3)^4$ in $(-2,3)$. (7M) CO2

(b) Find tan$x$ by Maclaurian's series upto the term containing x^5. (7M) CO2

(OR)

5. (a) Test for convergence of the series $\sum\frac{1}{n}\cos\left(\frac{1}{n}\right)$ (7M) CO2

(b) Test for convergence of the series $\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+\ldots\infty$ (7M) CO2

UNIT – III

6. (a) Find the half range sine series for $f(x)=x^2$ in $(0,\pi)$. (7M) CO3

(b) In a plane triangle find the maximum value of the function $\sin x\sin y\sin(x+y)$. (7M) CO3

(OR)

7. (a) Show that $\nabla^2(f(r))=f''(r)+\frac{2}{r}f'(r)$ (7M) CO3

(b) Find the directional derivative of xy^2+yz^3 at the point $(2,-1,1)$ in the direction of the normal to the surface $x\log z-y^2=-4$ at $(-1,2,1)$ (7M) CO3

UNIT – IV

8. (a) Find the rank of the matrix by reducing it to the normal form given (7M) CO4

$$A=\begin{bmatrix}0&1&2&3\\4&5&6&7\\1&1&1&1\end{bmatrix}$$

(b) For what value of k the equations $x+y+z=1,\ 2x+y+4z=k,\ 4x+y+10z=k^2$ has a solution and solve them completely in each case. (7M) CO4

(OR)

9. (a) Using Cayley-Hamiton theorem find the inverse of the matrix (7M) CO4

$$A=\begin{bmatrix}5&4&0\\1&2&0\\1&2&2\end{bmatrix}$$

(b) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to canonical form. (7M) CO4

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CE/EC/ME111(R20)